Affine definable $C^\infty G$ manifold structures in an o-minimal structure

1 Introduction

In this presentation, we consider an affine definable $C^\infty G$ manifold structure and its uniqueness of an affine definable C^r manifold when $1 \le r < \infty$.

Let $\mathcal{M} = (, +, \cdot, <, \dots)$ be an o-minimal expansion of the standard structure of the field of real numbers.

The following fact is known.

Theorem 1 *(1) If $0 \le r < \infty$, then every definable C^r manifold is affine (2005).*

(2) If $\mathcal{M}$ is exponential and G is an affine definable C^∞ group, then each compact definable $C^\infty G$ manifold is affine (1999).

We say that $\mathcal{M}$ is *polynomially bounded* if any definable function $f : \mathbb{R} \to \mathbb{R}$, there exist a positive integer N and $x_0 \in$ such that $|f(x)| < x^N$ for any $x > x_0$. If $e^x : \mathbb{R} \to \mathbb{R}$ is definable in $\mathcal{M}$, then $\mathcal{M}$ is exponential.

Theorem 2 (Miller 1994) *Every o-minimal expansion of the field of real numbers is either polynomially bounded or exponential.*

The following is an approximation theorem.

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Theorem 3 (2017) *If $0 \leq s < \infty$ and $\mathcal{M}$ admits C^∞ cell decomposition and exponential, then every definable C^sG map between affine definable $C^\infty G$ manifolds is approximated in the definable C^s topology by definable $C^\infty G$ maps.*

Our main result is the following.

Theorem 4 (2017) *Let X be an affine definable C^rG manifold and $\mathcal{M}$ admits C^∞ cell decomposition and exponential. If $1 \leqq r < \infty$ then, X admits a unique affine definable $C^\infty G$ manifold structure up to definable $C^\infty G$ diffeomorphism.*