Smooth One Fixed Point Actions of S_5 on Spheres

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Throughout the talk, manifolds and group actions on manifolds are ones in the smooth category. Let G be a finite group and M a compact manifold. A G-action on M is called a **one fixed point action** (or, **o.f.p. action**) if M^G consists of exactly one point. For existence of o.f.p. actions on spheres, the following necessary conditions were obtained.

1. (Laitinen–Traczyck) If $G \not\cong A_5$ or $n \neq 6$ then G does not admit o.f.p. actions on S^n such that $\dim S^H \leq 2$ ($\forall\ H \leq G$, $H \neq \{e\}$).

2. (Furuta) S^4 does not admit o.f.p. actions.

3. (DeMichelis, Kwasik–Schultz) For any $n \leq 5$, S^n does not admit o.f.p. actions.

4. (Borowiecka) S^8 does not admit o.f.p. actions of $SL(2,5)$.

5. S^9 does not admit o.f.p. actions of $SL(2,5)$.

On the other hand, we have found the following existence results of o.f.p. actions.

6. (Stein) $SL(2,5)$ has o.f.p. actions on S^7.
(Wall's surgery theory)

7. (Petrie) A_5 has o.f.p. actions on S^n for some n.
(G-surgery theory under strong gap condition)

8. (M.) For $n = 6$, 7, and $n \geq 9$, S^n has o.f.p. actions of A_5.
(G-surgery theory under gap condition)

9. (Laitinen–M.) Any Oliver group G has o.f.p. actions on spheres.

10. (Bak–M.) S^7 and S^8 have o.f.p. actions of A_5.
(G-surgery theory under weak gap condition)

In this talk, we discuss the next problem.

Problem 1. Is it valid that the sphere S^n of dimension n admits o.f.p. actions of $G = S_5$ if and only if n lies in $\{6\} \cup [10..12] \cup [14..\infty)$?

The following figures are helpful for our discussion.

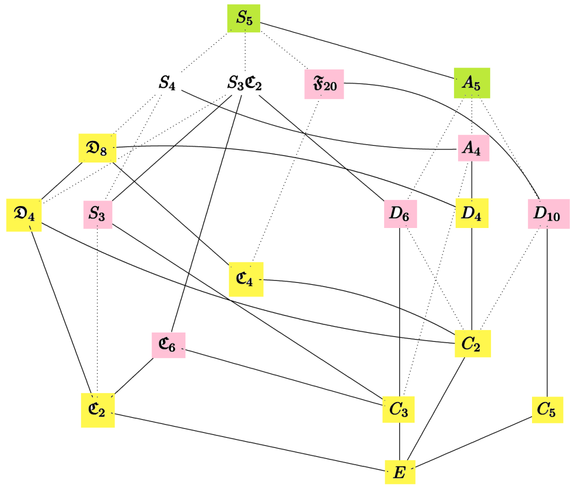

FIGURE 1. Subgroup lattice of S_5

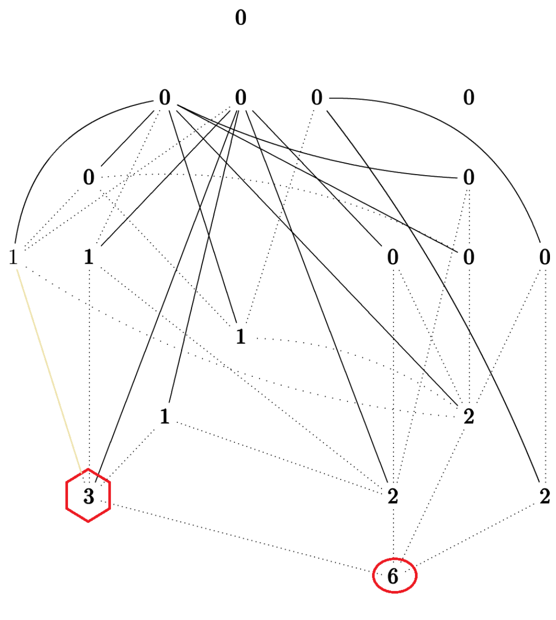

FIGURE 2. Fixed dimension of S_5-representation V_6